

Introductory Econometrics

Problem Set 5: Linear Regression and Binary Variables

1	Or
(1) ltsal	(1) ltsal
educ 0.0656*** (36.31)	educ 0.0656*** (0.0018)
exp 0.0134*** (6.97)	exp 0.0134*** (0.0019)
exp2 -0.0001 (-1.75)	exp2 -0.0001 (0.0000)
female -0.1381*** (-16.96)	female -0.1381*** (0.0081)
_cons 0.9971*** (23.67)	_cons 0.9971*** (0.0421)
<i>N</i> 4503	<i>N</i> 4503
adj. <i>R</i> ² 0.248	adj. <i>R</i> ² 0.248
<i>t</i> statistics in parentheses	Standard errors in parentheses
* <i>p</i> < 0.05, ** <i>p</i> < 0.01, *** <i>p</i> < 0.001	* <i>p</i> < 0.05, ** <i>p</i> < 0.01, *** <i>p</i> < 0.001

2

Step 1: hypothesis development

Test $H_0 : \delta = 0$ vs $H_1 : \delta \neq 0$

Step 2: test statistic

$$t = \frac{\hat{\delta} - 0}{\sqrt{\widehat{Var}(\hat{\delta})}} \sim_{H_0} t_{(n-(k+1))}$$

Degree of freedom: $n-k-1=4503-5=4498$

Step 3: Decision rule

$$|t_{obs}| > t_{(n-(k+1)), \alpha/2}$$

$|t_{obs}|=16.96 > 1.96$ so we reject H_0 at 5%.

(or could use the p-value decision rule. P-value < 0.0001, so we reject H_0 at 5%.)

Step 4: conclusion

In the sample, there is a significant difference between the wages of men and women, keeping fixed both education and experience.

Rmk: interpretation: a woman is paid less than a man by 13.81% on average, keeping fixed both education and experience.

	(1) Men ltsal	(2) Women ltsal
educ	0.061753*** (0.002595)	0.069242*** (0.002521)
exp	0.016969*** (0.002918)	0.010678*** (0.002549)
exp2	-0.000153* (0.000064)	-0.000010 (0.000057)
_cons	1.032341*** (0.060790)	0.813241*** (0.059275)
<i>N</i>	2179	2324
adj. <i>R</i> ²	0.212	0.247

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

4 Chow Test

Step 1: hypothesis development

H0: the regression function is the same for the two subsamples.

H1: the regression function is different for the two subsamples.

Step 2: test statistic

$$F = \frac{(RSS_0 - (RSS_1 + RSS_2))/(k + 1)}{(RSS_1 + RSS_2)/(n_1 - (k + 1) + n_2 - (k + 1))} \sim_{H_0} F_{(k+1, n-2(k+1))}$$

With RSS0 being the RSS of the pooled model. RSS1 and RSS2 are the RSSs of the subsamples. $n_1=2179$; $n_2=2324$.

Pooled model is

$$ltsal_i = \beta_0 + \beta_1 educ_i + \beta_2 Exp_i + \beta_3 Exp_i^2 + u_i$$

Degree of freedom: $k+1=4$; $n-2k-2=4503-8=4495$ ($k=3$)

Step 3: Decision rule

Fobs=74.04 > Fcritical value=2.37; so we reject H₀ at 5%.

Step 4: conclusion

There are significant differences in impacts of education and experience on salary between men and women.

	(1) Men ltsal	(2) Women ltsal	(1) all ltsal
educ	0.061753*** (0.002595)	0.069242*** (0.002521)	0.061753*** (0.002556)
exp	0.016969*** (0.002918)	0.010678*** (0.002549)	0.016969*** (0.002875)
exp2	-0.000153* (0.000064)	-0.000010 (0.000057)	-0.000153* (0.000063)
female			-0.219100** (0.084871)
seduc			0.007489* (0.003616)
sexp			-0.006290 (0.003867)
sexp2			0.000144 (0.000085)
_cons	1.032341*** (0.060790)	0.813241*** (0.059275)	1.032341*** (0.059892)
<i>N</i>	2179	2324	4503
adj. <i>R</i> ²	0.212	0.247	0.249

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The coefficients of the non-interacted variables match the estimated coefficients obtained using the sample of men (female=0).

The coefficients of the interacted variables match the estimated coefficients obtained using the sample of women minus those using the sample of men (female=1-female=0).

6 The Chow test in 4 is equivalent to a F-test here (exactly the same!)

test female=seduc=sexp=sexp2=0

Step 1: hypothesis development

H0: the coefficients before female=seduc=sexp=sexp2=0

H1: at least one of them is non zero.

Step 2: test statistic

$$\frac{(RSS^C - RSS^{NC})/q}{RSS^{NC}/(n - (k + 1))} \sim_{H_0} F_{q, n - (k + 1)}$$

With RSS^C being the RSS of the restricted model and RSS^{NC} being the RSS of the unrestricted model.

restricted model is

$$ltsal_i = \beta_0 + \beta_1 educ_i + \beta_2 Exp_i + \beta_3 Exp_i^2 + u_i$$

Degree of freedom: $q=4$; $n-k-1=4503-8=4495$ ($k=7$)

Step 3: Decision rule

$F_{obs}=74.04 > F_{critical\ value}=2.37$; so we reject H_0 at 5%.

Step 4: conclusion

There are significant differences in impacts of education and experience on salary between men and women.