Innovation Incentives under Frictional Matching Along the Global Value Chain

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Abstract

This paper develops a 1-m frictional positive assortative matching (PAM) model along the global value chain. We show the existence of multiple equilibria where a supplier could be matched to different headquarters, but at different stages on the value chain. We show that innovation increases a party's productivity and thus increases the probability of being matched with a value chain with higher profits. However, the higher the total productivity of the value chain, the fewer headquarters/suppliers there are; innovation then reduces the probability of being matched at all. By distorting the matching probability, frictions can further distort firms' innovation incentives. The incentives to innovate are determined by these two countervailing forces. Thus, this paper provides a framework that can explain heterogeneity in innovativeness within industrial networks, why high-tech firms often prefer to outsource only low-tech activities, and why many start-ups voluntarily adopt low-tech strategies precisely in a high-tech economic environment.

Key words: Innovation, global value chain, positive assortative matching, imperfect information.

1 Introduction

A significant amount of international economic activities involves production along global value chains. Successful company usually has a headquarter in a country (a region) and multiple suppliers around the world. These suppliers produce goods that is used in different stages on the value chain. What is the matching patterns between the headquarter and these suppliers? Are the patterns heteogenous at different stages? What would be the implications for innovation incentives? In this paper, we develop a model with features of frictional positive assortative matching to answer these questions in a decentralized market equilibrium.

We develop a 1-headquarter-m-stage positive assortative matching (PAM) model to study the relationship between innovation and productivity along global value chains on the firm level. Firms could be categorized as headquarters (downstream firm) or suppliers (upstream firms). Production on each value chain involves inputs from both the headquarter and suppliers. Each value chain is composed of a headquarter and multiple positions for suppliers. There are two dimensions of productivity rankings. On a given value chain, we could rank the positions for suppliers by productivity and call them stages on the chain. We could also rank the value chains by the headquarter's productivity. Suppliers are matched to value chains according to their productivity.

We build on Grossman and Helpman (2005) to capture the fact that the production of one piece of goods involves inputs from both downstream and upstream firms. We shut down the price distortion channel and omit the difference in fixed costs between vertical integration and outsourcing in their model. In contrast, we introduce a quality-adjusted utility function as in Antràs and Chor (2013) to capture heterogenous firm productivity.

We explicitly discuss the matching between downstream and upstream firms. To start with, we adapt a simplified static version of the Shimer and Smith (2000) matching model. We preserve the same result that one agent's accepting set is a neighborhood around her own productivity. i.e., the matching area is of band shape. However, the mechanism behind frictional matching is different. In their dynamic model, frictions raise from intertemporal inefficiency. Match creation is time consuming and each agent faces the tradeoff between better future match and immediate match. Here, in our static model, frictions come from imperfect information about partner's productivity. Agents face the tradeoff between paying higher cost and meeting a larger set of partners.

We conduct two extensions of the benchmark model to show that the band shape matching area is robust to different assumptions. In the first extension, we introduce competition among agents. For a given agent, instead of only one agent who could be matched with her, we allow for the existence of a set of agents who are eligible to make the match. Competition reduces the agent's matching rate. This reduces the size of the matching area but the matching area remains band shaped. Each agent is still matched to a partner within a neighborhood around her own productivity but the range of the neighborhood narrows as competition increases. In the second extension, we allow each agent to update their expectation about the their partners to be matched. As matching only occurs when both sides agree, meetings in which one agent is willing to be matched but the other agent is unwilling to be matched is inefficient. By updating expectation about partners, an agent could save resources by avoiding inefficient meetings. We show that the band-shape matching area preserves in this extension as well. However, updating expectation benefits high productivity firms more than low productivity firms because it excludes more inefficient meetings for high productivity firms and thus saves more resources for them.

Then, we extend this static 1-1 matching model to a static 1-m matching model which describes PAM along value chains. In order to generate a sequence of production stages increasing in technology (productivity) endogenously, we borrow the sequential form production function in Antras and Chor (2013). However, instead of emphasizing on physical sequentiality as in their paper, we emphasize on technology sequentiality in the paper. We introduce an index to rank stages by technology.

Finally, we move to show that our model provides novel types of incentives for innovation. As to be discussed below, the complete model could explain several interesting observations in reality.

First, PAM along value chains could explain the stage-quality tradeoff we observe in the real world. For a given supplier, it could be matched to a high stage on a value chain with a headquarter of low productivity. It could also be matched to a value chain with a headquarter of high productivity, but at a low stage. As shown in Chen (2014), suppliers of the same productivity could be matched to headquarters of different productivity at different production stages on the value chain, which she calls a paradox in upgrading on the global value chain. Firms that ally with dragon's heads are trapped at the bottom while firms that ally with Guerilla Investors raced to the top. Government in developing countries usually enforce policies aiming at participation in high technology value chain. The result is, however, counterproductive. Local firms start at top but are pushed to the bottom. This seemingly counterintuitive result could be explained by the PAM model in which a supplier is matched to a producer with close productivity. In our 1-headquarter-m-stage value chain PAM model, the producer's productivity level contains two parts: the headquarter's productivity and the stage's productivity.

Second, PAM along value chains could explain a novel source of positive incentives for innovation. In the classical literature of innovation, the reason for innovation is that it improves a firm's own productivity. Innovation benefits the firm in an additional way in our PAM model where profit is affected by a firm's own productivity as well as the productivity of its matched value chain. Under perfect information, an increase in productivity of the supplier will result in a match with a headquarter of higher productivity than before. Therefore, innovation improves the probability of being matched with a value chain of higher productivity. Profits are increased by a part generated by increase in the firm's own productivity plus a part due to higher productivity of the headquarter and other suppliers on the new value chain.

Third, frictional matching could explain potential losses from innovation. In reality, instead of having strong innovation incentives, low productivity firms are afraid of innovation. If they stay at current technology level, their chance to be matched with a value chain is high. If they move up, in addition to the R&D cost incurred, they will also have lower chance of getting matched with a value chain. The empirical results are mixed. Dachs, Ebersberger, Kinkel and Som (2014) investigated the effects of production offshoring on the innovation activities of manufacturing firms in the home country. They find a positive relationship. Mihalache et al. (2012) reveal an inverted U-shaped relationship between offshoring and firm innovativeness. Karpaty and Tingvall (2011) find for Swedish multinational firms that offshoring has a negative effect on R&D intensity at home. The feature of frictional matching in our model could explain the negative incentives. Matching area in PAM model is a line. However, with imperfect information , the matching area is of band shape under distributions with fat tail region for low productivity, such as Pareto Distribution. The probability of being matched decreases as the firm's own productivity increases. Therefore, innovation decreases the probability of being matched and is not always good to the firm.

This paper relates to mainly two streams of literature. One literature is global value chain (global supply chain). There is a large literature in this topic, including Grossman and Helpman (2005), Costinot, Vogel and Wang (2013), Antras and Chor (2013), Baldwin and Venables (2013) etc. Recently, papers focus on firm level studies, including Dragusanu (2014) and Alfaro, Antras, Chor and Conconi (2015). Our benchmark model is built upon Grossman and Helpman (2005) where the production of one piece of goods involves upstream and downstream firms. As our research focus is different from their paper, we simplified several components of their model. We extend the simplified model in order to discuss frictional matching between upstream and downstream firms along the value chain. We also discuss the implications for innovation incentives. Another paper close to this paper is Dragusanu (2014). Both paper discuss imperfect PAM pattern. However, our paper are different in the sense that we have different reasons for the frictional matching.

In Dragusanu (2014), she shows that firms at different physical production stages have different strength of PAM because of the degree of holdup problem. In our paper, the strength of PAM is affected by the productivity of firms and this because productivity affects the range of productivity of value chains that could be matched with the firm. Our results are align with Dragusanu (2014) as many studies show that downstreamness and productivity are positively related.

Another related literature is the two side heterogenous matching models started from Becker (1973) and developed by lots of researches including Shimer and Smith (2000), Smith (2006) and Atakan(2006). Their model is a 1-1 dynamic model. We first simplify their settings to a static one and show that the frictional PAM patterns in their model could be preserved in our static setting. Then we extend the model to a 1-m matching model. We show the existence of multiple equilibria in the 1-m model.

Next section gives the model set-up. In section 3, we describe our matching environment and derive the matching area. In section 4, we give the 1-m extension. In section 5, we discuss different innovation incentives. Section 6 concludes.

2 Model

We begin with a simple version of the model where the production of one product involves two firms, an upstream firm and a downstream firm. In section 4, we extend the model to a 1-m value chain model to capture the fact that in real world one product contains several components and different upstream firms participate in the production of it.

The basic model is build on Grossman and Helpman (2005) where we have a downstream firm to produce the final goods and an upstream firm to produce the component. Total profit is composed of inputs from both the downstream and upstream firms. They share the total profit with a share of ω_j and $1-\omega_j$. j represents organization modes, including vertical integration and outsourcing. Different from their model, we allow for heterogenous productivity of downstream firms and upstream firms. Total profit depends on the productivity of both sides. In addition, we explicitly discuss how downstream and upstream firms are matched to each other.

2.1 Productivity, Demand and Profits

There is a continuum of industries $v \in [0,1]$. In each industry v, firms differ in their productivity level $s(v)$. Assume the productivity level follows Pareto Distribution, i.e. $s(v) \sim s(v)^{-a^i}$ ($\Pr(s(v) > s) = s^{-a^i}$) on $[1, \infty]$, where a^i is country specific.

Final goods enter a CES utility function of the representative consumer. Like in Acemoglu et al. (2003), final goods differs in two senses. First is the standard variety dimension $y(v)$. Second is the specificity dimension $s(v)$ which presents productivity level here. We can think the specificity dimension $s(v)$ as the technology level of the product. A high tech product gives consumers more utility than a low tech one. Or we can think of the utility function as a quality-adjusted utility function as in Antràs and Chor (2013). Different from Acemoglu et al. (2003), we consider a continuum of heterogenous firms, so the variety dimension depends on v and $s(v)$.

$$u = [\int_0^1 \int_1^\infty s(v)^{\frac{1}{\epsilon}} y(v, s(v))^{\frac{\epsilon-1}{\epsilon}} ds(v) dv]^{\frac{\epsilon}{\epsilon-1}}$$

This specific functional form insures that one unit of increase in $s(v)$ maps into one unit of increase in total profits. The linearity simplifies calculation and gives us the well-known demand function,

$$y(v, s(v)) = s(v)\frac{E}{P}p(v, s(v))^{-\epsilon}$$

where $P = [\int_0^1 \int_1^\infty s(v)p(v, s(v))^{1-\epsilon} ds(v)dv]^{\frac{1}{1-\epsilon}}$. E denotes expenditure.
We assume that revenue and cost are shared proportionally by both sides. This is different from the existing literature but it helps to highlight our mechanism due to frictional matching between upstream and downstream firms. In existing literature, revenue is shared proportionally but costs are paid by the side with the possession of the property. As downstream firms possess capital and upstream firms possess labor, the capital rent is paid by downstream firms completely and the wage is paid by upstream firms completely. This results in a distorted price relative to standard profit maximization problem. We shut down this channel because we are not going to study the relationship between capital-labor intensity and firm's organization structure choice. Under our assumption, there is no conflicts in setting price between upstream and downstream firms. Mathematically, it means that the maximization of ω_j share of total profit is the same as the maximization of the total profit. Therefore, we have the standard profit maximization problem,

$$\max_{p(v,s(v))} p(v, s(v))y(v, s(v)) - w^i y(v, s(v))$$

s.t. $y(v, s(v)) = s(v)\frac{E}{P}p(v, s(v))^{-\epsilon}$
This gives us the optimal price and profit.

$$p(v, s(v)) = \frac{\epsilon}{\epsilon - 1}w^i$$

$$\pi(v, s(v)) = (\frac{\epsilon - 1}{\epsilon})^\epsilon \frac{1}{\epsilon - 1}(w^i)^{1-\epsilon}s(v) \equiv \xi(w^i)^{1-\epsilon}s(v)$$

where $\xi = (\frac{\epsilon-1}{\epsilon})^\epsilon \frac{1}{\epsilon-1}$.
We have the result that $\pi(v, s(v))$ is linear in $s(v)$ which will simplify the calculation.

2.2 Upstream and Downstream Firm Productivity

To produce one unit of goods, we need two firms: an upstream firm and a downstream firm. The productivity level $s(v)$ in the profit function depends on both the upstream productivity $s_u(v)$ and downstream productivity $s_d(v)$. Industries are symmetric, so we omit the industry index v. We have the following profit function,

$$\pi(s_u, s_d) = \xi(w^i)^{1-\epsilon} f(s_u, s_d)$$

where $\xi = (\frac{\epsilon-1}{\epsilon})^{\epsilon} \frac{1}{\epsilon-1}$.

Downstream firm get ω_j share of the profits,

$$\pi(s_u, s_d) = \omega_j \xi(w^i)^{1-\epsilon} f(s_u, s_d)$$

Upstream firm get $1 - \omega_j$ share of the profits,

$$\pi(s_u, s_d) = (1 - \omega_j)\xi(w^i)^{1-\epsilon} f(s_u, s_d)$$

It is not costless to reach the combined technology level $f(s_u, s_d)$. Downstream firms and upstream firms pay heterogenous costs for search and match. This will be discussed in detail in next section.

2.3 Timing

Timing of the events in the complete model is described as the followings.

1. **Entry**

   Upstream firms and downstream firms enter with fixed costs. Upstream $f_u w_i$; downstream $f_d w_i$. At this stage, firms don't know the exact productivity of the partner firm they will meet. At a later stage, they can choose the optimal set of firms to meet with. At this stage, they compare the expected profits and decide to enter the market or not.

2. **Choosing Organization Mode**

   If the downstream firm chooses to enter the market, it chooses the optimal organization modes based on it's own productivity s_d. As emphasized in the recent literature of global value chains started from Antràs (2003), the main difference between outsourcing and vertical integration is the share of revenue. Vertical integration gives downstream firm a higher share of profits due to the hold-up problem of incomplete contracts. i.e. $\omega_V > \omega_N$ where ω_V is the share of profits of downstream firms for vertical integration and ω_N is the share of profits for outsourcing (non-integration). We follow Antràs and Chor (2013) and Alfaro et al. (2015) and focus on this key difference. We omit the difference in fixed costs between vertical integration and outsourcing.

3. **Innovation**

   Firms can innovate and improve their productivity. Downstream firm chooses its innovation effort level z. By paying a cost $d(z)$, s_d can be improved to zs_d in expectation. Similarly, the productivity of upstream firms, s_u, can be improved to es_u in expectation by paying a cost $c(e)$.

4. **Search and Match**

   Upstream and downstream firms search and match. Our PAM model is based on Shimer and Smith (2000), with adaptions to the upstream and downstream firm matching environment. Firms can choose the range of partners under imperfect information. One side of firms cannot perfectly observe the productivity of the other side. They can only know the range of firms' productivity. This information is not costless. They need to pay a specific cost. We discuss the modeling set-ups in detail in next section.

5. **Production of Intermediate Goods**

   Upstream firms produce the intermediate goods according to the demand of final goods. One component is assembled to one final good and thus the amount of intermediate goods equals the amount of final goods and is determined by the standard profit maximization under monopolistic competition.

6. **Production of Final good**

   Downstream firms produce the final goods according to the demand function.

7. **Bargaining**

   If there is production, upstream firm will get $1 - \omega_j$ ($j = V$ or N) share of profits and downstream firm will get ω_j share of profits. Otherwise, both parties get 0.

We solve it by backward induction. This provides a full picture of how firm-level matching, organization modes and innovation interacts with each other. Production of intermediate and final goods are already discussed in section 2.1. In the following sections, we discuss matching, organization modes and innovation.

3 Firm-Level Matching

We discuss the match between upstream and downstream firms in detail in this section. We propose a novel type of matching among heterogenous agents under imperfect information. Instead of being matched to one partner or being matched to all partners as in the existing literature, we allow firms to choose an optimal set of partners. In equilibrium, within a certain range of productivity, firms are matched together randomly.

The market is divided into different segments: $[1, 1+\Delta]$, $[1+\Delta, 1+2\Delta]$, $[1+2\Delta, 1+3\Delta]$...$[1+(k-1)\Delta, 1+k\Delta]$...where Δ is exogenous. We can think of these segments as fairs or communities where firms meet with each other. The market segments are ranked by the range of productivity of firms who participate in. We call these market segments fairs henceforth. Assume that there is a large number of potential entrants. So within each segment, firm productivity follows the Pareto Distribution. When they meet (before production), firms don't know the exact productivity level of the partner in a fair. They only know the range of productivity for a given fair. Attending a fair is not costless, firms need to pay a cost $g(k)$ for the kth fair.

Rational firms only attend fairs that give them positive profits in expectation. As a result, there is an optimal fair participation strategy for each firm. In equilibrium, matching only occur when both the firm and its desirable partner attend the same fair. This gives us a matching area within which firms are matched to each other.

3.1 Production Function

To get analytical results, we assume a Cobb-Douglas production function that contains both sides' productivity. That is $f(s_u, s_d) = s_u^{1-\alpha} s_d^{\alpha}$. The profits function becomes,

$$\pi^d(s_u, s_d) = \omega_j \xi (w^i)^{1-\epsilon} s_u^{1-\alpha} s_d^{\alpha} \equiv c s_u^{1-\alpha} s_d^{\alpha}$$

for downstream firms, and

$$\pi^u(s_u, s_d) = (1-\omega_j) \xi (w^i)^{1-\epsilon} s_u^{1-\alpha} s_d^{\alpha} \equiv c_\omega c s_u^{1-\alpha} s_d^{\alpha}$$

for upstream firms.
where $c = \omega_j \xi (w^i)^{1-\epsilon}$ and $c_\omega = \frac{\omega_j}{1-\omega_j}$.
We focus on the firm-level matching in this section, so we shut down the effects of organization modes (ω_j) and wage (w^i).

3.2 Optimal Fair Strategy

Rational firms attend fairs that give them positive expected profits. When expected profit is non-monotonic in the rank of fairs (productivity level), we have Interval Matching: firms choose to meet partners within an interval of productivity. The following lemma states the results. Take a downstream firm as an example.

Lemma 1. *A firm will attend the kth fair $[1+(k-1)\Delta, 1+k\Delta]$ iff it can get nonnegative expected return in the fair, i.e.,* $\mathbb{E}\pi(k) = \int_{1+(k-1)\Delta}^{1+k\Delta} c s^{1-\alpha} s_d^{\alpha} p(s) ds - g(k) \geq 0$

(Interval Matching) If $\mathbb{E}\pi(k)$ is non-monotonic, firms attend fairs in a certain interval, $[k_1, k_2], [k_3, k_4]$....
($\underline{k}$ could be 1 and $\overline{k}$ could be ∞.)

We can get symmetric results for upstream firms.

Assume that the productivity is of Pareto Distribution. Assume that the cost is proportional to technology distance between the two participants, i.e., $g(k) = (k - s_d)^2$, where s_d is productivity level of a given downstream firm, we have the result that firms attend fairs within $[\underline{k}, \overline{k}]$.

Lemma 2. *Under Pareto Distribution, when $g(k) = (k - s_d)^2$, we have $\frac{\partial \mathbb{E}\pi(k)}{\partial k} > 0$ for small k and $\frac{\partial \mathbb{E}\pi(k)}{\partial k} < 0$ for larger k. So we have Interval Matching. Firms attend fairs within $[\underline{k}, \overline{k}]$.*

3.3 Matching Area

Combining Lemma 1 and Lemma 2, we know that the optimal fair strategy under costs that are proportional to productivity distance is to attend fairs in an interval according to their own productivity $[\underline{s_u}(s_d), \overline{s_u}(s_d)]$ $([\underline{s_d}(s_u), \overline{s_d}(s_u)])$. Firms have two decisions to make: the upper bound and lower bound. This results in a band-shaped Matching Area.

Take a downstream firm s_d for example, the profit maximization problem now is,

$$\max_{\underline{s_u}, \overline{s_u}} \int_{\underline{s_u}}^{\overline{s_u}} [c s_d^{\alpha} s^{1-\alpha} p(s) - (s - s_d)^2] ds$$

F.O.C. ($\overline{s_u}$)

$$c a s_d^{\alpha} \overline{s_u}^{-a-\alpha} = (\overline{s_u} - s_d)^2 \tag{1}$$

F.O.C. ($\underline{s_u}$)

$$c a s_d^{\alpha} \underline{s_u}^{-a-\alpha} = (\underline{s_u} - s_d)^2 \tag{2}$$

Similarly, we have the following optimal conditions for a upstream firm,

$$c c_{\omega} a s_u^{1-\alpha} \overline{s_d}^{-a-1+\alpha} = (\overline{s_d} - s_u)^2 \tag{3}$$

$$c c_{\omega} a s_u^{1-\alpha} \underline{s_d}^{-a-1+\alpha} = (\underline{s_d} - s_u)^2 \tag{4}$$

Proposition 1. *Matching Area Under Costs Proportional to Productivity Distance*

Under Pareto Distribution, when $g(k) = (k - s)^2$, the Matching Area is shaped by the equation system (5)-(8) which is a band.
Moreover, it narrows as productivity increases.

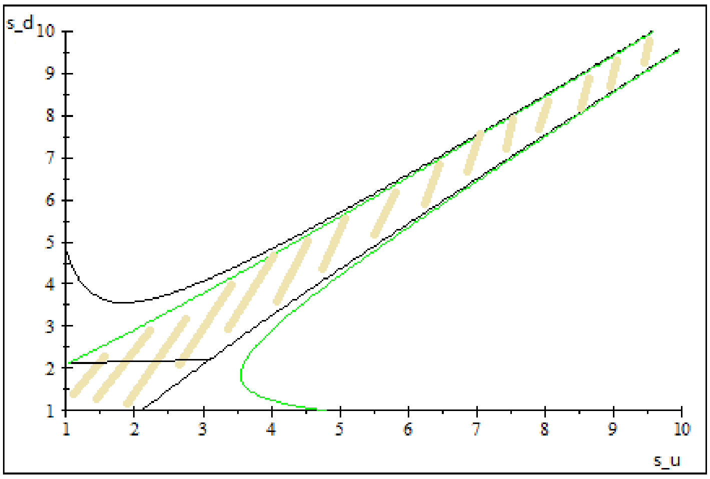


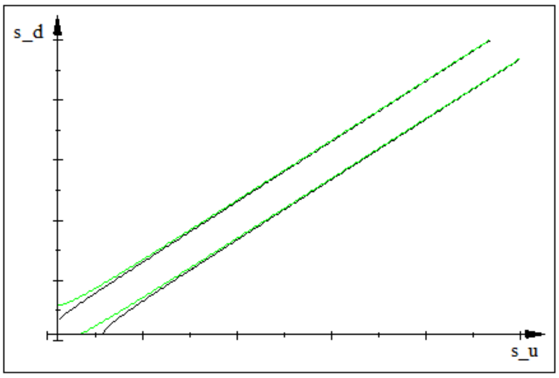
Figure 1: numerical solution $a = 1.2$, $c = 50$, $c_\omega = 1$, $\alpha = 0.5$.

The assumption that the matching costs is proportional to productivity distance generates a source of positive assortative matching. For a given firm, the matched partners are those whose productivity is within an neighborhood around the firm's own productivity. In equlibrium, the matching area is a band. Expected profits is affected by mass of potential partners as well as the firm's productivity. To have the same expected profits, higher productivity firms choose a smaller range of partners. So the band narrows as the productivity increases. Figure 1 shows the matching area.

The narrowing pattern for firms of high productivity depends on the heavy-tail distribution. Under Pareto Distribution, the mass for high productivity firms is smaller than the mass for low productivity firms. It might not be true for other distributions. For example,if we have uniform distribution for matching probability, the matching are is still of band shape. However, the width of the band is constant. Figure 2 shows the cases for parameters (a) not belong to Pareto Distribution. The mathematical reason can be seen in the proof of proposition 1.

4 Matching Along the Value Chain

Global value chain is a heat topic in international trade. However, there is very few existing researches on firm-level matching along global value chains. In this section, we extend the benchmark matching model to a 1-m value chain model. The main result that downstream and upstream firms match to each other in a semi positive assortative way is the same in the extension. More importantly, it generates an additional tradeoff for downstream firms – the tradeoff between the position on the



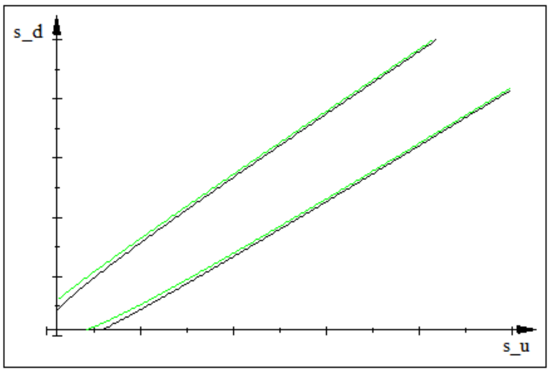


Figure 2: numerical solution $c = 10$, $c_\omega = 1$, $\alpha = 0.5$. 1nd graph $a = 0$ and 2nd graph $a = -0.5$

value chain and the position among the value chains.

4.1 Sequential Production Increasing in Technology Endogenously

First, we show how to endogenously generate a sequence of production stages increasing in technology (productivity). The main idea is to put an increasing sequence of weights of each stage to rank the production stages by technology. We borrow the sequential form production function in Antras and Chor (2013). However, instead of emphasizing on physical sequentiality as in their paper, we emphasize on technology sequentiality. Thus, we introduce an index $R(j)$, which is increasing in stages (j), to rank stages by technology.

$$y(\theta) = \theta(\int_0^1 x(j)^\alpha R(j)dj)^{1/\alpha} \tag{5}$$

where θ is the productivity of the headquarter. Equivalently, it is the quality of the value chain. Within each value chain, there is a sequence of stages $[0, 1]$. $x(j)$ is the input at stage j. $R(j)$ is the technology rank of stage j which is increasing in j. α represents the complementarity between stages. Each value chain has two dimensions. One is its own quality θ and the other is each stage's technology level $R(j)$. The quality-adjusted input of each stage contains two elements. One is the "real" input $x(j)$ and one is the stage's "nominal" rank $R(j)$.

A firm maximizes the net output,

$$\max_{\{x_j\}_{j\in[0,1]}} \theta(\int_0^1 x(j)^\alpha R(j)dj)^{1/\alpha} - \int_0^1 x(j)dj$$

where $\int_0^1 x(j)dj$ is the cost of the total production. This gives us an endogenous increasing sequence of production stages. i.e. we will have $x(j) > x(j')$ if $j > j'$.

To see this, look at the F.O.C.s

$$x(j)^{1-\alpha} = \theta R(j)(\int_0^1 x(j)^\alpha dj)^{\frac{\alpha-1}{\alpha}}$$

By assumption, $R(j) > R(j')$ if $j > j'$. As $1-\alpha > 0$, $x(j) > x(j')$ if $j > j'$. So equation (5) can generate a sequence of production stages endogenously increasing in technology.

4.2 Firm-Level Match

Then, we embed the matching model in section 3 into the value chain framework. Instead of choosing the optimal level of $x(j)$ by paying a cost $x(j)$ for $R(j)x(j)^\alpha$, the headquarter firm (downstream firm) can only search and match with suppliers (upstream firms) $x(j)$. i.e., it can only choose an optimal set of partners $x(j)$'s under imperfect information. Now, the optimal set depends on the technology rank of the value chain.

4.2.1 Optimal Fair Strategy

We assume the firm productivity distribution for each stage $p(j)$'s are independent. The headquarter firm's expected profit of attending kth fair for $x(j)$ is

$$\mathbb{E}\pi(k) = c\theta(\int_0^1 \int_{1+(k-1)\Delta}^{1+k\Delta} x(j)^\alpha R(j)p(x(j))dx(j)dj)^{1/\alpha} - \int_0^1 g(\frac{k}{R(j)}, j)dj$$

For headquarter θ, the perceived technology level to match is $\frac{x(j)}{R(j)}$ instead of $x(j)$. So the matching cost is $g(\frac{k}{R(j)}, j)$ in place of $g(k, j)$.
For supplier $x(j)$, she will share the profits of the jth stage. As the output of jth stage is $\theta x(j)^\alpha R(j)$, $\theta R(j)$ is the perceived technology level to match, rather than θ. So the matching cost is $g(R(j)k, j)$ in place of $g(k, j)$.
The expected profit of attending kth fair for supplier $x(j)$ is

$$\mathbb{E}\pi(k) = cc_\omega \int_{1+(k-1)\Delta}^{1+k\Delta} \theta x(j)^\alpha R(j)p(\theta)d\theta - g(R(j)k, j)$$

By the same logic as in lemma 1 and lemma 2, we could get the interval matching result for headquarters and suppliers.

4.2.2 Matching Area

As in the benchmark model, we assume that the firm productivity follows Pareto Distribution and the matching costs are distance proportional. The shape of the matching area at a particular stage j is the same as in the 1-1 matching model. However, the matching area moves for different stages along the chain.

We first solve the optimal condition for headquarter and suppliers. This gives us the matching area.

Headquarter's problem,

$$\mathbb{E}\pi(k) = c\theta(\int_0^1 \int_{\underline{x(j)}}^{\overline{x(j)}} x(j)^\alpha R(j)p(x(j))dx(j)dj)^{1/\alpha} - \int_0^1 \int_{\underline{x(j)}/R(j)}^{\overline{x(j)}/R(j)} (\theta - x(j))^2 dx(j)dj$$

F.O.C. ($\overline{x(j)}$)

$$ca\frac{1}{\alpha}(\int_0^1 \int_{\underline{x(j)}}^{\overline{x(j)}} R(j)x(j)^\alpha p(x(j))dx(j)dj)^{\frac{\alpha-1}{\alpha}} \theta R(j)^2 = \overline{x(j)}^{a+1-\alpha}(\theta - \frac{\overline{x(j)}}{R(j)})^2 \quad (6)$$

F.O.C. ($\underline{x(j)}$)

$$ca\frac{1}{\alpha}(\int_0^1 \int_{\underline{x(j)}}^{\overline{x(j)}} R(j)x(j)^\alpha p(x(j))dx(j)dj)^{\frac{\alpha-1}{\alpha}} \theta R(j)^2 = \underline{x(j)}^{a+1-\alpha}(\theta - \frac{\underline{x(j)}}{R(j)})^2 \quad (7)$$

Supplier's problem,

$$\mathbb{E}\pi(k) = cc_\omega x(j)^\alpha R(j) \int_{\underline{\theta}}^{\overline{\theta}} \theta p(\theta)d\theta - \int_{\underline{\theta}R(j)}^{\overline{\theta}R(j)} (\theta - x(j))^2 d\theta$$

F.O.C. ($\overline{\theta}$)

$$cac_\omega x(j)^\alpha = \overline{\theta}^a(\overline{\theta}R(j) - x(j))^2 \quad (8)$$

F.O.C. ($\underline{\theta}$)

$$cac_\omega x(j)^\alpha = \underline{\theta}^a(\underline{\theta}R(j) - x(j))^2 \quad (9)$$

The above optimal conditions give us the matching area. For a given stage, its shape is the same as in our benchmark case.

Proposition 2. *Matching Area*
Under Pareto Distribution of productivity, the matching area is constrained by (6) - (9) for distance proportional costs. For each stage j, the matching area, we have band-shaped Matching Area.

4.3 Stage-Quality Trade-off

A more interesting finding is the trade-off between position along the value chain and position among the value chains we observed in reality. We call this Stage-Quality Trade-off.

Proposition 3. *Stage-Quality Trade-off*
For same θ, $x(j)$ increases as $R(j)$ increases. The headquarter endogenously increases quality demand as the technology rank of stages increases.
For same $x(j)$, $R(j)$ decreases as θ increases. The supplier will be pushed to a low position on the value chain (in terms of technology) if she's going to match to a better headquarter. This is the stage-quality trade-off observed in real world.

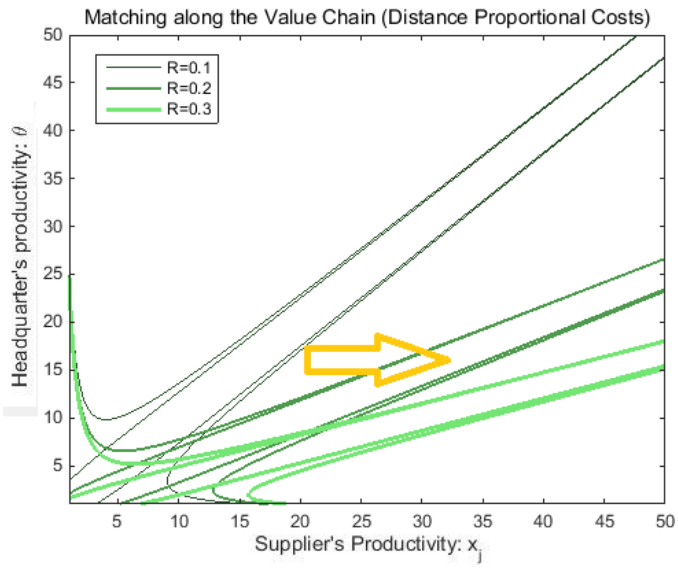


Figure 3: numerical solution $a = 1.2$, $c = 20$,$c_\omega = 1$, $\alpha = 0.8$. The lighter and the thick the line, the higher the technology rank of the stage.

The next proposition states a testable prediction from the model.

Proposition 4. *The cross*

$$x(j) = \xi_1 R(j)^{\frac{a}{(1+a)(1+a+1-\alpha)-\alpha}}$$

$$\theta = \xi_2 R(j)^{\frac{-a-2}{(1+a)(1+a+1-\alpha)-\alpha}}$$

where ξ_1, ξ_2 are constants.
Keep the productivity of the headquarter fixed, as $R(j)$ increases (the technology rank increase), the productivity of matched suppliers increases. Keep the productivity of the supplier fixed, as $R(j)$ increases, the productivity of matched headquarter decreases.

This proposition implies that if the productivity range of the suppliers are fixed, as the productivity of the headquarters increases, the stages where it could find matched suppliers becomes lower. This captures that in real world, high tech firms only outsource basic processes in their value chain to other suppliers.

In developing countries/regions, government always has incentives to attract high-tech companies. Usually, in exchange, they offer to produce some basic tasks in local firms. Their wish is to upgrade positions of local firms in the global economy. However, experience shows that it doesn't work well for corporation with high-tech firms. Instead of moving up the value chain, local firms are pushed down to low-tech stages, even lower than the original position. For example, in the case study of Chen (2014), local firms have the technology to produce semiconductor components. However, they are only asked for assembly on the high-tech firm's value chain.

Paradoxically, when matched with middle-tech headquarter, the suppliers preserved their position to produce components, even are asked to participate in R&D stage. Though counter-intuitive at first glance, our simple static model can explain this phenomenon.

Instead of the traditional one-dimension way to think about the global value chain, we point out that global value chain is of two dimensions. One is the level of different stages while the other is the level of the chain itself. This results in multiple equilibria. A supplier could be matched to a stage on a certain chain. Or equivalently, she could be matched to another stage on a different chain. As a result, some policy-enforced outcomes might be unstable (e.g. low-tech supplier, high position, high-tech producer).

5 Innovation Incentives

The classic trade-off for innovation is between profits and costs. PAM matching could result in a more subtle cost for innovation. If a firm innovates "too much", it will have small probability to be matched with a partner.

The model is the same as before. However, now firms will choose to innovate (increase productivity) or not. If they innovate, their productivity will increase. If they do not innovate, their productivity level stays the same. To highlight the cost of innovation due to lower matching probability, we shut down the innovation costs in classic literature. We show that even without costs to produce innovation, low matching probability could still generate incentives to not innovate. We use backward induction to solve the model. The profit function after innovation has the same form, but at a higher productivity level. So we will have the same optimal fair strategy and Matching Area, at a higher productivity level.

Take a downstream firm as an example, if the firm choose to innovate, its productivity becomes $s_d^+ = s_d + \Delta$. It will meet upstream firms in $[\underline{s_u^+}, \overline{s_u^+}] = [\underline{s_u^*}(s_d + \Delta), \overline{s_u^*}(s_d + \Delta)]$.

The profit maximization problem becomes,

$$\pi_{innov} = \int_{\underline{s_u^*}(s_d+\Delta)}^{\overline{s_u^*}(s_d+\Delta)} [c(s_d + \Delta)^\alpha s^{1-\alpha} p(s) - (s - s_d - \Delta)^2] ds$$

where $\underline{s_u^*}(s_d + \Delta)$ and $\overline{s_u^*}(s_d + \Delta)$ is defined by (1) and (2).

If the firm choose to not innovate, its profit is,

$$\pi_{notinnov} = \int_{\underline{s_u^*}(s_d)}^{\overline{s_u^*}(s_d)} [c s_d^\alpha s^{1-\alpha} p(s) - (s - s_d)^2] ds$$

where $\underline{s_u^*}(s_d)$ and $\overline{s_u^*}(s_d)$ is defined by (1) and (2).

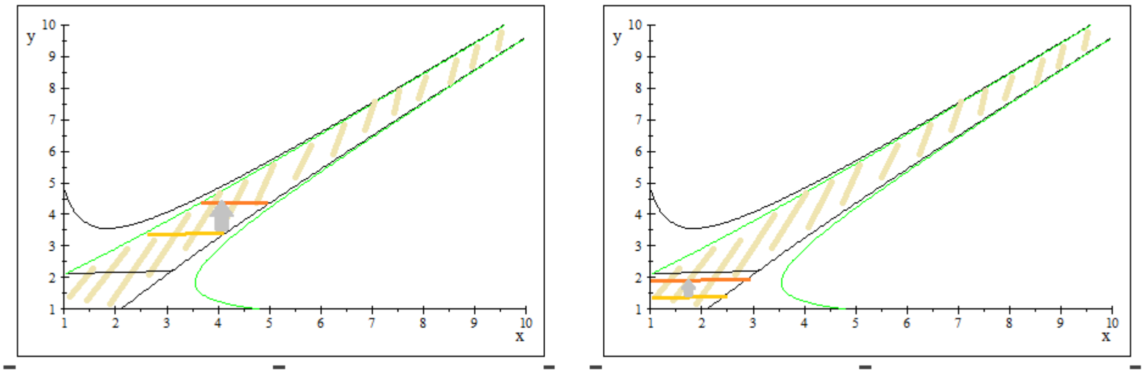


Figure 4: numerical solution $a = 1.5$, $c_1 = c_2 = 2.5$,$\omega = 0.5$.

The firm will choose to innovate iff

$$\pi_{innov} > \pi_{notinnov}$$

We can see the benefit of meeting better partners (the second term) increases firm's innovation incentives. However, the cost of narrowing matching range (the third term) decreases firm's innovation incentives.

We can get similar results for the low technology range. The following proposition gives the full result.

Proposition 5. *Matching Affects Innovation*

(general, doesn't depend on Pareto Distribution)

When after innovation technology level is within the low range ($s_d^+ \in [1, \underline{s_d}(1)]$), firms have more incentive to innovate under this limited matching area. This is because by innovation, they can meet better partners.

When after innovation technology level is within the high range ($s_d^+ \in [\underline{s_d}(1), s_d^*]$), the result is ambiguous. On the one hand, the chance to meet better partners increases their innovation incentives. On the other hand, the afraid of narrowing matching range decreases their innovation incentives.

As firms are in symmetric position, we have mirror result for upstream firms.

6 Conclusion

In this paper, we develop a matching model along value chains which has rich features. Firstly, it features imperfect PAM patterns started from Shimer and Smith (2000). The PAM provides innovation incentives that are different from those in the classical literature. Secondly, we develop a 1-m value chain matching model. This gives us multiple equilibria. A given supplier could be matched to different headquarters, but at different stages. Finally, our model predicts a negative relationship between imperfectness and firm productivity under fat tail distribution.

The matching area narrows as the firm productivity increases. This generates a source of negative innovation incentive.

The next step could be using data to test our model. However, as there is very few access to firm level matching data, we could only do it in an circuitous way. We could use data on intermediate goods to see if the quality of inputs and quality of outputs are positively related and if the variation decreases with firm productivity. A further step could be analyzing what affects level and variation of input-output quality relationship, in addition to firm productivity. For example, we could see if firm structure (integrated or not), financial constraint and access to intermediates (so to reduce frictions) plays a role in matching.

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A Proofs for 1-1 Matching

Proof of Lemma 2

$$\frac{\partial E\pi(k)}{\partial k} = cs_d^{\alpha}(1+k\Delta)^{1-\alpha}p(1+k\Delta)\Delta - cs_d^{\alpha}(1+(k-1)\Delta)^{1-\alpha}p(1+(k-1)\Delta)\Delta - g'(k)$$

$$= cs_d^{\alpha}(1+k\Delta)^{1-\alpha}p(1+k\Delta)\Delta - cs_d^{\alpha}(1+(k-1)\Delta)^{1-\alpha}p(1+(k-1)\Delta)\Delta - 2(k-s_d)$$

Under Pareto distribution, we have

$$\frac{\partial E\pi(k)}{\partial k} = cas_d^{\alpha}\Delta[(1+k\Delta)^{-\alpha-a} - (1+(k-1)\Delta)^{-\alpha-a}] + 2(s_d - k)$$

The first term $cas_d^{\alpha}\Delta[(1+k\Delta)^{-\alpha-a} - (1+(k-1)\Delta)^{-\alpha-a}] < 0$

The second term $2(s_d - k) > 0$ when k is small and $2(s_d - k) < 0$ when k is large.

So we have $\frac{\partial E\pi(k)}{\partial k} > 0$ when k is small and $\frac{\partial E\pi(k)}{\partial k} < 0$ when k is large. Q.E.D.

Proof of Proposition 1

The Matching Area is shaped by equation (1)-(4). Or equivalently, it's determined by the following equation system.

$$cay^{\alpha}x^{-a-\alpha} = (x-y)^2 \tag{A.1}$$

$$cc_{\omega}ax^{1-\alpha}y^{-a-1+\alpha} = (x-y)^2 \tag{A.2}$$

We prove proposition 1 in the following steps.

1. Band Shape

   Equation (A.1) is equivalent to

   $$x^{a+\alpha+2} - 2yx^{a+\alpha+1} + y^2x^{a+\alpha} - cay^{\alpha} = 0$$

   This has two real roots when y is not too small. This means the one side matching area generated by equation (A.1) is band shape. Similarly, equation (A.2) also generates a band shape matching area.

   The solutions of equation (A.1) satisfy

   $$x_{\text{big1}} = y + \sqrt{cay^{\alpha}x_{\text{big1}}^{-a-\alpha}}$$

   $$x_{\text{small1}} = y - \sqrt{cay^{\alpha}x_{\text{small1}}^{-a-\alpha}}$$

   The solutions of equation (A.2) satisfy

   $$x_{\text{big2}} = y + \sqrt{cc_{\omega}ax_{\text{big2}}^{1-\alpha}y^{-a-1+\alpha}}$$

   $$x_{\text{small2}} = y - \sqrt{cc_{\omega}ax_{\text{small2}}^{1-\alpha}y^{-a-1+\alpha}}$$

   This implies that $x_{\text{big1}} > x_{\text{small2}}$ and $x_{\text{big2}} > x_{\text{small1}}$. So the two bands always have common spaces. This means the two-side matching area is still a band. Further more, this implies that the matching area is governed by two upper bounds, not two lower bounds.

2. s_u increases as s_d increases.
   Equation (A.1) is equivalent to

$$ca(\frac{y}{x})^{\alpha} = x^a(x-y)^2$$

   An increase in y implies an increase in x. However, x,y don't increase at the same magnitude. Which increases more depends on the value of a.
   Under Pareto Distribution, $a > 1$. This implies the following pattern.
   When $x > y$, because $(x-y)^2$ increases as x increases, x increases less than y. When $x < y$, due to the same logic, x increases more than y. So $(x-y)^2$ decreases as y increases, regardless of $x > y$ or $x < y$.
   This pattern holds for all $a > 0$.When $a = 0$, x,y increase at the same magnitude.When $a < 0$, we have the opposite result as to $a > 0$. So $(x-y)^2$ increases as y increases.
   We can get same result from (A.2).

3. Under Pareto Distribution, the distance of matched firms narrows as s_u and s_d increases.
   Step 1 and 2 implies the matching area narrows as s_u and s_d increases. Q.E.D.

B Proofs for Matching along the Value Chain

Proof of Proposition 2

For distance proportional costs, the matching area is governed by,

$$cac_\omega x(j)^{\alpha} = \overline{\theta}^a(\overline{\theta}R(j) - x(j))^2 \tag{B.1}$$

$$cac_\omega x(j)^{\alpha} = \underline{\theta}^a(\underline{\theta}R(j) - x(j))^2 \tag{B.2}$$

$$b_2\theta R(j)^2 = \overline{x(j)}^{a+1-\alpha}(\theta - \frac{\overline{x(j)}}{R(j)})^2 \tag{B.3}$$

$$b_3\theta R(j)^2 = \underline{x(j)}^{a+1-\alpha}(\theta - \frac{\underline{x(j)}}{R(j)})^2 \tag{B.4}$$

where $b_2 = ca(\int_0^1 \int_{\underline{x(j)}}^{\overline{x(j)}} x(j)^{\alpha}R(j)p(x(j))dx(j)dj)^{\frac{\alpha-1}{\alpha}}$
$b_3 = ca\frac{1}{\alpha}(\int_0^1 \int_{\underline{x(j)}}^{\overline{x(j)}} R(j)x(j)^{\alpha}p(x(j))dx(j)dj)^{\frac{\alpha-1}{\alpha}}$
Take (B.3) for example, define $f(\theta, R) = \theta^a(\theta R - x)^2$

$$\frac{\partial f}{\partial \theta} = a\theta^{a-1}(\theta R - x)^2 + 2\theta^a R(\theta R - x)$$

$$\frac{\partial f}{\partial R} = 2\theta^{a+1}(\theta R - x)$$

$$\frac{\partial R}{\partial \theta} = -\frac{\partial f/\partial \theta}{\partial f/\partial R} = -\frac{a(\theta R - x) + 2\theta R}{2\theta^2}$$

As we have proved in Proposition 1, the matching area is governed by (B.3), not (B.4), $\theta R - x > 0$. So, $\frac{\partial R}{\partial \theta} < 0$.
Similarly, we could prove $\frac{\partial R}{\partial x} > 0$ for (B.1).
So we have $\overline{\theta'} < \overline{\theta}$ if $R(j') > R(j)$ for a given $x(j)$ and $\overline{x(j)'} > \overline{x(j)}$ if $R(j') > R(j)$ for a given θ *within* the matching area.